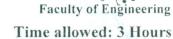


## Math. & Eng. Physics Dept.

## Preparatory year Students



Course Code: BAS 1011

Mathematics (1)

## Answer the following questions (Full mark 130 marks)

- 1. (a) [5 marks] If  $f(x) = \sin 2x$  and  $g(x) = \tan^{-1} x$ . Write (g/f)(x) and  $(f \circ g)(x)$  in its simplest form and compute their domains.
  - (b) [10 marks] Find each of the following limits
    - (i)  $\lim_{x \to 0} (\csc x \cot x)$ ,
- (ii)  $\lim_{x \to 0^{+}} (1 + 4 \sinh x)^{2/x}$ .
- (c) [10 marks] Find the domain, range and discuss the symmetry of the functions  $f(x) = 2^{-x^2}$  and  $g(x) = \tanh^{-1} x$ . Sketch the graph of these functions and then prove that  $\tanh^{-1} x = \frac{1}{2} \ln \left( \frac{1+x}{1-x} \right)$  on its domain.
- (d) [10 marks] If  $y = (\sinh^{-1} x)^2$ , prove that  $(1+x^2)y'' + xy' = 2$ , hence, or otherwise, deduce that  $(1+x^2)y^{(n+2)} + (2n+1)xy^{(n+1)} + n^2y^{(n)} = 0$ .
- 2. (a) [5 marks] Solve for x the equation:  $2 \ln x^2 = (\ln x)^2 + 3$ .
  - (b) [10 marks] Evaluate
    - (i)  $\frac{dy}{dx}$  if  $(\cosh x)^y + y = 3^{\tan^{-1} x}$ , (ii)  $\frac{d^2y}{dx^2}$  if  $y = \sin^{-1} t$  and  $x = \operatorname{sech}^{-1} t$ .
  - (c) [10 marks] Find Taylor's expansion of the function  $f(x) = e^{3x-3}$  about the point x = 1.
  - (d) [10 marks] Show that the function  $f(x) = e^{\sin x} + x$ , defined on the interval  $[0, 2\pi]$ , satisfies the hypotheses of the mean value theorem for differentiation. Then, find all points c satisfying the conclusion of the theorem.

## ion 3 [30 marks]

Find the coefficient of  $x^n$  in the expansion of  $\frac{1}{1-x} + \frac{2x}{(1-2x)^2}$ , Is it possible to find the

coefficient of 
$$\left(\frac{3}{4}\right)^n$$
? Why.

[10 marks]

3-b) If x = 2 is a root of the equation  $x^3 + bx^2 - 11x + 30 = 0$ , find the partial fraction

decomposition of 
$$\frac{6x^2 - 13x - 13}{x^3 + bx^2 - 11x + 30}$$

[10 marks]

**3-c)** If 
$$A = \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}$$
, use the mathematical induction to prove that  $A^n = \begin{bmatrix} 3^n & 3^n - 2^n \\ 0 & 2^n \end{bmatrix}$ ,  $n \ge 1$ .

Then find the eigenvalues and the corresponding eigenvectors of the matrix  $A^n$ . [10 marks]

# Question 4 [40 mark]

Given the matrices

$$A = \begin{bmatrix} -1 & 0 & -2 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}, \quad C = \begin{bmatrix} -2 \\ 2 \\ 0 \end{bmatrix}, \quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

1) Find  $A^{-1}$ 

[5 marks]

2) Find the diagonal matrix **D** such that  $\mathbf{D} = \mathbf{A}^{-1} \mathbf{B} \mathbf{A}$ 

[5 marks]

3) Evaluate  $D^{-1}$ ,  $D^{10}$ ,  $\det(A^{-1}B^{-1}A)$ 

[5 marks]

4) Solve the equation BAX - AC = 0

[5 marks]

- 5) State "True" or "False". Noting that you should explain the answer:
  - a) The matrix B is non-singular.

[5 marks]

b) The homogeneous system BX = 0 has non-trivial solutions.

[5 marks]

c) For any positive integer m,  $B^m = AD^mA^{-1}$ .

[5 marks]

d) The rank of the augmented matrix  $\begin{bmatrix} D & C \end{bmatrix}$  is the same as of the augmented matrix

$$[A \mid C]$$
 and equals 2.

[5 marks]