



Mansoura University

Course Code: BAS 1011

Math. & Eng. Physics Dept.

Preparatory year Students

Mathematics (1)



Faculty of Engineering

Time allowed: 3 Hours

Answer the following questions (Full mark 130 marks)

1. (a) [5 marks] If $f(x) = \sin 2x$ and $g(x) = \tan^{-1} x$. Write $(g/f)(x)$ and $(f \circ g)(x)$ in its simplest form and compute their domains.
 - (b) [10 marks] Find each of the following limits
 - (i) $\lim_{x \rightarrow 0} (\csc x - \cot x)$,
 - (ii) $\lim_{x \rightarrow 0^+} (1 + 4 \sinh x)^{2/x}$.
 - (c) [10 marks] Find the domain, range and discuss the symmetry of the functions $f(x) = 2^{-x^2}$ and $g(x) = \tanh^{-1} x$. Sketch the graph of these functions and then prove that $\tanh^{-1} x = \frac{1}{2} \ln \left(\frac{1+x}{1-x} \right)$ on its domain.
 - (d) [10 marks] If $y = (\sinh^{-1} x)^2$, prove that $(1+x^2)y'' + x y' = 2$, hence, or otherwise, deduce that $(1+x^2)y^{(n+2)} + (2n+1)xy^{(n+1)} + n^2 y^{(n)} = 0$.
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2. (a) [5 marks] Solve for x the equation: $2 \ln x^2 = (\ln x)^2 + 3$.
 - (b) [10 marks] Evaluate
 - (i) $\frac{dy}{dx}$ if $(\cosh x)^y + y = 3^{\tan^{-1} x}$,
 - (ii) $\frac{d^2 y}{dx^2}$ if $y = \sin^{-1} t$ and $x = \operatorname{sech}^{-1} t$.
 - (c) [10 marks] Find Taylor's expansion of the function $f(x) = e^{3x-3}$ about the point $x = 1$.
 - (d) [10 marks] Show that the function $f(x) = e^{\sin x} + x$, defined on the interval $[0, 2\pi]$, satisfies the hypotheses of the mean value theorem for differentiation. Then, find all points c satisfying the conclusion of the theorem.

Question 3 [30 marks]

Find the coefficient of x^n in the expansion of $\frac{1}{1-x} + \frac{2x}{(1-2x)^2}$. Is it possible to find the

coefficient of $\left(\frac{3}{4}\right)^n$? Why.

[10 marks]

3-b) If $x = 2$ is a root of the equation $x^3 + bx^2 - 11x + 30 = 0$, find the partial fraction

decomposition of $\frac{6x^2 - 13x - 13}{x^3 + bx^2 - 11x + 30}$

[10 marks]

3-c) If $A = \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}$, use the mathematical induction to prove that $A^n = \begin{bmatrix} 3^n & 3^n - 2^n \\ 0 & 2^n \end{bmatrix}$, $n \geq 1$.

Then find the *eigenvalues* and the corresponding *eigenvectors* of the matrix A^n . [10 marks]

Question 4 [40 mark]

Given the matrices

$$A = \begin{bmatrix} -1 & 0 & -2 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}, \quad C = \begin{bmatrix} -2 \\ 2 \\ 0 \end{bmatrix}, \quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

1) Find A^{-1} [5 marks]

2) Find the diagonal matrix D such that $D = A^{-1} B A$ [5 marks]

3) Evaluate D^{-1} , D^{10} , $\det(A^{-1} B^{-1} A)$ [5 marks]

4) Solve the equation $BAX - AC = 0$ [5 marks]

5) State "True" or "False". Noting that you should explain the answer:

a) The matrix B is non-singular. [5 marks]

b) The homogeneous system $BX = 0$ has non-trivial solutions. [5 marks]

c) For any positive integer m , $B^m = AD^m A^{-1}$. [5 marks]

d) The rank of the augmented matrix $[D | C]$ is the same as of the augmented matrix

$[A | C]$ and equals 2.

[5 marks]